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The flow of rarefied gases between two parallel plates

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Abstract. The flow rates of hydrogen, helium, air, carbon dioxide and water vapour between two parallel optically flat glass surfaces have been measured. The distance between the two surfaces was of the order of $10\ \mu\text{m}$ and the flow experiments were carried out with pressures in front of and behind the channels of the order of 10 torr and 0.1 torr respectively. A flow equation is presented which describes the flow of the gases and the vapour through the channels.

1. Introduction

The flow of rarefied gases through fine channels usually lies within the limits of viscous flow, with slip on the one hand and diffusive flow on the other. The exact mode of flow at any channel cross section is dependent on the ratio of the number of molecular-wall collisions to the number of intermolecular collisions. When the ratio is much larger than unity diffusive flow is observed, when the ratio is much smaller than unity viscous flow is observed and when the ratio is of the same order as unity a transition region exists which is strongly dependent on the slip flow. An equation for the diffusive flow of gases between two parallel plates was first derived by Knudsen (1909) by considering the momentum transfer between the channel surfaces and the gas phase. von Smoluchowski (1910) showed that there were two errors in the momentum transfer method, and went on to derive a shape factor to describe the variation in the flow equation from Knudsen's equation by using the directional distribution method first used by Knudsen for long circular capillaries. Hiby and Pahl (1952) have extended the directional distribution method to include the effects due to long molecular paths and anisotropic spatial distribution resulting from intermolecular collisions for circular capillaries and parallel plates. Their work agrees well with the experimental results of Knudsen and of Gaede (1913) for parallel plates and for capillaries when the Knudsen number α , the ratio of the channel width or diameter to the molecular mean free path, is less than 0.01. In the viscous flow region the usual theoretical analysis leads to an equation representing Poiseuille flow, together with an additional flow due to the first layer of gas molecules 'slipping' over the channel surfaces. Various theoretical methods have been used to derive the slip term (see Fryer 1966), but no term explains the variation in the slip coefficient observed experimentally by Millikan (1923), Stacey (1923) and others.

Weber (1954), Scott and Dullien (1962) and Fryer (1966) wrote the flow equation as the sum of the three flow terms, viscous flow, slip flow and diffusive flow, but with each term multiplied by a function representing the fraction of the molecules involved in each particular mode of flow. The equations have a minimum in the specific flow as a function of α in the region where the contribution of the slip term is of the same order as the contribution of the diffusive term.

The experiments described in this work were carried out with the aim of obtaining reliable quantitative results for the flow of hydrogen, helium, air, argon, carbon dioxide and water vapour between parallel plates, which could then be used in a theoretical study of the slip flow term and to show any surface effects which might occur for water vapour.

2. Experimental method

The parallel-plate channels were made by separating two optically flat surfaces of crown-glass blocks with a thin metal-foil spacer. The channel length and breadth were both between 10 mm and 40 mm and the channel widths, which were measured by observing Edser-Butler interference fringes, were between $7\ \mu\text{m}$ and $40\ \mu\text{m}$. The channel width was measured *in situ* daily, since from day to day it exhibited a random variation of the order

of $0.4 \mu\text{m}$. Further, at the beginning and end of every set of experiments the channel width found by the optical method was checked against the value obtained from flow experiments carried out at atmospheric pressure. At atmospheric pressure the flow could be assumed to be entirely viscous since then α is of the order of 100 for air.

The apparatus was constructed of glass, except for the vacuum vessel housing the channel, which was made of brass. A diagram of the apparatus is given in figure 1.

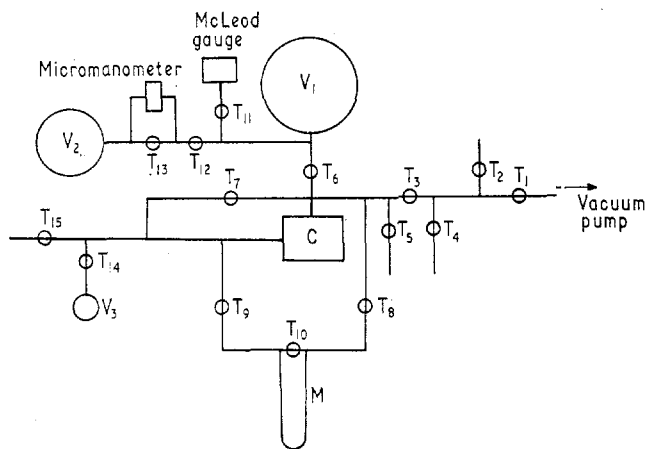


Figure 1. The apparatus.

Before any flow experiments were carried out pumping and leak detecting continued until the combined effect of leaks and desorption (outgassing) was less than 2×10^{-5} torr l s^{-1} for every part of the apparatus, which corresponds to 1% of the lowest flow rate of air measured. With this condition being satisfied, the rates of flow of hydrogen, helium, air, argon, carbon dioxide and water vapour were measured for different differential pressures across the channel and different channel widths by using the following procedures.

For the gases the rate of flow was measured by allowing the gas, flowing at equilibrium through the channel C, to collect in a large known volume V_1 on the low-pressure side of C. The rate of change in pressure in V_1 , $\delta P/\delta t$, was measured and hence the mass rate of flow could be calculated. In order to simplify the theoretical treatment of the flow the pressure behind the channel was kept as low as possible, the limit being set in most cases by the accuracy with which $\delta P/\delta t$ could be measured. By using a micromanometer (supplied by Furness Controls Ltd) pressure changes of 0.02 torr in V_1 could be measured to within 2.5% and thus for narrow channels the back pressure could be kept under 0.1 torr; if a lower pressure change had been used the error in $\delta P/\delta t$ would increase proportionally. For channels of widths greater than $25 \mu\text{m}$ back pressures of up to 0.2 torr were recorded which were due to the limitations of the pumping system and could not be avoided with the apparatus used. Further, in order to keep the experimental error as low as possible the time lag between the increase in pressure immediately behind the channel and the increase in pressure in the micromanometer had to be much smaller than the time of measurement. To ensure that this was so it was calculated that the diameter of the tube leading from the vacuum vessel housing the channel to the volume V_1 must be greater than 25 mm; the diameter used was 30 mm.

The gas to be used was supplied to the apparatus already dried, at the required constant pressure (between 4 torr and 20 torr), through tap T_{15} with taps T_7 and T_{14} closed. The gases were supplied in trade cylinders (except in the case of air) and the pressure kept constant by means of a needle valve and a Cartesian manostat (supplied by Edwards High Vacuum Ltd). The gases were allowed to flow through the channel into the low-pressure side of the apparatus which was continuously evacuated by the pumps. Then after approximately 15 min, when the pressure across the channel was constant (measured by the

Apiezon oil manometer), taps T_{11} , T_3 and T_{13} were closed, in that order, and the rate of increase in the pressure of V_1 measured by the micromanometer. The McLeod gauge was used to measure the initial back pressure and to check the micromanometer calibration periodically. The McLeod gauge could not be used to measure $\delta P/\delta t$ since its absolute sensitivity decreases as the pressure increases and since careful readings take much time. Blank experiments were performed as a final check that leaks and desorption did not contribute to the results. The main sources of error in the experiments were due to the calibration of the micromanometer and the value of the volume V_1 ; however, the error for every result was less than 4%.

The method used for the gases could not be used for vapours, since the amount of vapour adsorbed on glass or metal surfaces increases rapidly with increasing pressure. However, the flow of water vapour was measured under equilibrium conditions by collecting and weighing the vapour in a liquid-nitrogen trap on the low-pressure side of the channel. The trap was a small removable U tube and the procedure employed was such that there was no chance of any vapour being lost from the trap during the removal and weighing. The U tube, having a constriction in each limb, was sealed onto the outlets from taps T_4 and T_5 with Apiezon wax, and the whole apparatus was then evacuated until leaks and desorption were less than 2×10^{-5} torr 1 s^{-1} . The required differential pressure was then set up across the channel by allowing water to evaporate away in the small flask V_3 , which was kept at a constant temperature, and by continuously evacuating the low-pressure side of C. When equilibrium between adsorption and desorption had been obtained in the low-pressure side (usually after a few hours) tap T_3 was closed and liquid nitrogen placed around the U tube to freeze out the water vapour. After a known time interval of the order of a few hours the U-tube constrictions were sealed and the amount of water vapour collected found from a series of weighings. Careful blank experiments were carried out to check that adsorption and desorption of water vapour on and off the glass apparatus did not contribute to the amount of water vapour collected.

All the experiments were carried out at a channel temperature of 20 °C; Macpherson (1965) showed that for similar channels the mass rate of flow Q is proportional to $(M/T)^{1/2}$ when all other parameters are constant.

3. The results

Since the pressure behind the channel was less than 2% of the pressure in front for all the experiments, the variable parameters for each gas were the channel dimensions, the differential pressure ΔP and Q . It was shown during the work that Q was proportional to the ratio of the channel breadth to the channel length, b/l , when all other parameters are constant; therefore all the results are presented in terms of the variable G where $G = Q(T/M)^{1/2}l/b$. Unfortunately a comprehensive table of the results cannot be included in the space available but all the results are available (see Roberts 1968). A typical set of results for a single channel width is shown in figure 2. For simplicity and to help discussion, the results are shown as graphs of G against h at a constant ΔP and are given in figure 3 for $\Delta P = 7.72$ torr (120 mm oil). The choice of $\Delta P = 7.72$ torr is arbitrary; similar graphs could have been shown for other values of ΔP . Comparison between the different gases and vapours is now possible, since the small variation in h , shown in figure 2, can be neglected. For h less than 10 μm the values of G_p for all the gases and for water vapour, except for helium, were the same within the experimental error of 4%. For h of the order of 7 μm helium shows values of G_p 15% higher than those for the heavier gases; this will be discussed in § 5. The variation of G_p with α_p (the Knudsen number on the high-pressure side) is also discussed in § 5, where it is shown that all the gases and water vapour are represented by a single function $f(\alpha_p)$, where $f(\alpha_p)$ is dependent on G_p , h and the viscosity coefficient η .

4. Theory

As already mentioned, in a theoretical treatment of the flow of gases through fine channels the flow can be divided into three régimes: viscous flow, slip flow and diffusive flow. Adequate formulae for both the viscous and diffusive régimes have been developed

for circular capillaries and parallel plates, but no formula exists which fully describes the slip flow régime. Fryer (1966) derived a single equation for gas flow through circular capillaries applicable for flow rates extending from viscous flow to diffusive flow, but his equations and methods are not easily adapted to describe the flow between two parallel plates.

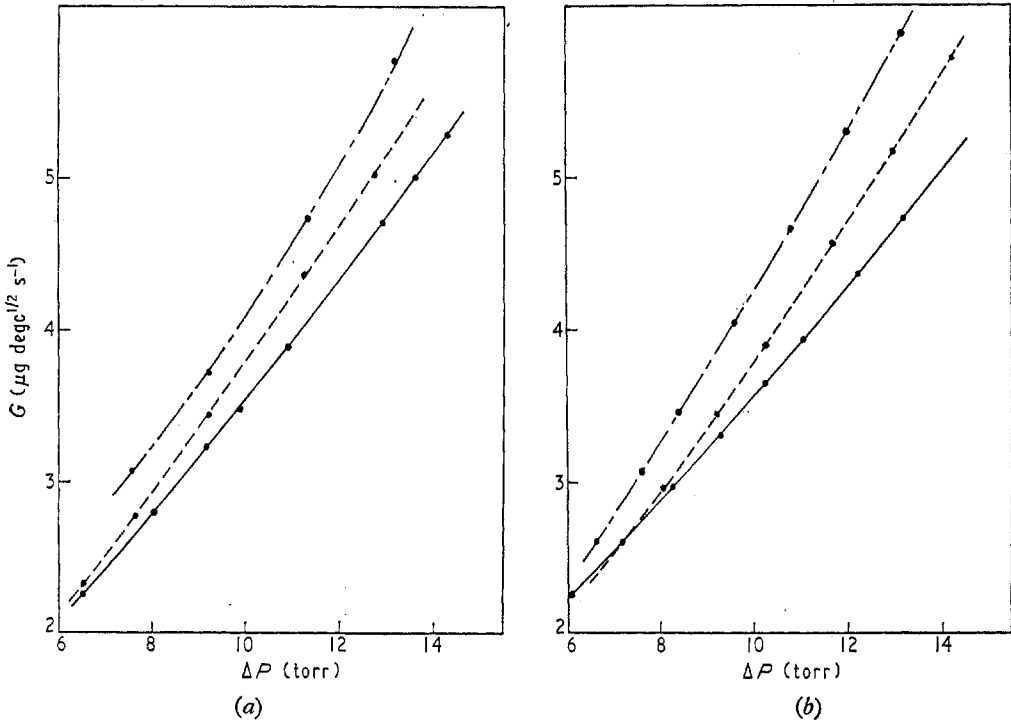


Figure 2. (a) Variation of G with ΔP : ——— hydrogen ($h = 14.7 \mu\text{m}$); - - - - air ($h = 14.6 \mu\text{m}$); — · — · — water vapour ($h = 14.8 \mu\text{m}$). (b) Variation of G with ΔP : ——— helium ($h = 14.9 \mu\text{m}$); - - - - argon ($h = 14.7 \mu\text{m}$); — · — · — carbon dioxide ($h = 14.9 \mu\text{m}$).

To simplify the problem of calculating the gas rate of flow between two parallel plates when the differential pressure is of the same order as the mean pressure, an element of channel length δl will be considered where the differential pressure δP across the element is much less than the mean pressure of the element. For such an element the usual derivation of viscous flow with slip (similar to that of Kennard 1938, p. 294) leads to the equation

$$Q = -\frac{bh^3}{12\eta} \frac{MP}{RT} \frac{\delta P}{\delta l} + \frac{hbPM}{RT} U_0$$

where U_0 is the slip velocity and η is the shear viscosity. In the following treatment of the above equation the Knudsen number α (or in Fryer's terminology P/P_1) is used; thus by using the equation $\eta = 0.499\rho\bar{c}\lambda$ where \bar{c} is the molecular mean velocity, λ is the molecular mean free path and ρ is the density, a parameter β can be defined where

$$\beta = \lambda P = 1.257\eta \left(\frac{RT}{M} \right)^{1/2}.$$

The flow equation then becomes

$$Q = -\frac{\beta bh}{12} \left(\frac{\pi M}{RT} \right)^{1/2} \alpha \frac{\delta \alpha}{\delta l} + \frac{\beta bM}{RT} U_0 \alpha \quad (1)$$

where

$$\alpha = h/\lambda = hP/\beta.$$

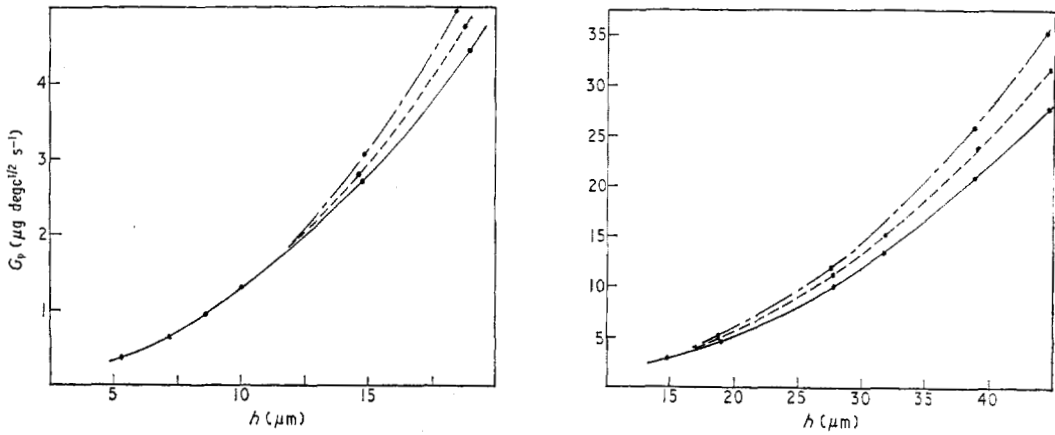


Figure 3(a). Variation of G_p with h for hydrogen —, air - - - and water vapour — · — · —; $\Delta P = 7.72$ torr.

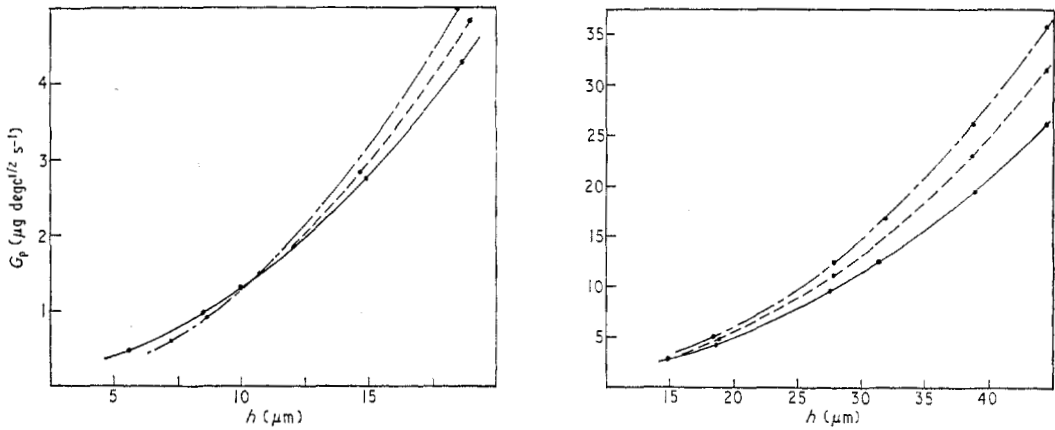


Figure 3(b). Variation of G_p with h for helium —, argon - - - and carbon dioxide — · — · —; $\Delta P = 7.72$ torr.

4.1. The slip term

The slip velocity U_0 which appears in equation (1) is usually derived by considering the rate of change of momentum of molecules colliding with the wall. Let us consider a gas flowing over a plane surface with a flow velocity v which varies in a direction perpendicular to the surface (i.e. along the z axis). Maxwell (see Kennard 1938, pp. 295-6) assumed the velocity gradient to be uniform above a certain value of z ; thus the slip velocity U_0 is obtained by extending the uniform velocity profile to $z = 0$. His value of U_0 was then obtained from the momentum transfer equation

$$f\left(\frac{1}{2}\eta\frac{dv}{dz} + \frac{1}{4}\rho\bar{c}U_0\right) = \eta\frac{dv}{dz}$$

where f is the fraction of molecules diffusely reflected at the wall. But in the treatment Maxwell assumed that the viscous phase extends right up to the wall and that the slip velocity is the velocity of the edge of the phase relative to the wall. The assumption might be true in the case of specular reflection at the wall since molecules approaching and leaving

the wall possess a mean velocity in the direction of flow. However, in the case of diffuse reflection, provided diffusive flow is neglected (see § 4.3), molecules leaving the wall have zero flow velocities; hence the viscous phase does not extend right up to the wall for such molecules. Fraser (1931) and Massey and Burhop (1952) have shown that all gases are reflected diffusely according to the cosine law, except for the reflections of hydrogen and helium off cleaved surfaces where slight specular reflection is observed for glancing angles less than 6° . Therefore in the following treatment only diffusive reflection at the wall will be considered.

In order to satisfy the condition that molecules join the viscous phase at a finite distance from the wall, it will be assumed that molecules leaving a surface and travelling towards another parallel surface possess viscous and slip velocities between the plane where they join the viscous phase and their next surface collision. Hence the slip velocity is assumed to be the average velocity in the direction of flow that molecules which have just left a wall attain when they join the viscous phase. The value of the slip velocity may therefore be different from the value derived by Maxwell, since Maxwell derives U_0 from the momentum transfer when molecules collide with the wall.

The slip velocity will therefore be derived as follows. We take the element δl and consider the momentum transfer when molecules join the viscous phase by attaining the slip velocity U_0 . Since molecules leaving a wall have zero flow velocities, then as shown by Present and Pollard (1948) the rate at which these molecules gain momentum from the viscous phase per unit area parallel to the wall is $\frac{1}{2}nm\bar{c}U_0$, where n and m are the molecular density and mass. But this can be equated with the shearing stress in the gas (see Kennard 1938, p. 296); hence, after substituting for $\eta = \frac{1}{2}nm\bar{c}\lambda$, the slip velocity becomes

$$U_0 = 2\lambda \frac{dv}{dz}$$

and equation (1) becomes

$$Q = -\frac{\beta bh}{2} \left(\frac{\pi M}{2RT}\right)^{1/2} \left(\frac{\alpha}{6} + 2\right) \frac{\delta\alpha}{\delta l} \quad (2)$$

where the slip term is 2 and the viscous term is $\alpha/6$.

Although the value of U_0 just derived is twice the value derived by Kennard (1938, p. 296), the derivation differs from that given by Kennard only in that the $\frac{1}{2}\eta dv/dz$ term in Kennard's equation has been neglected. In Kennard's equation the term represents the momentum transfer due to the assumed uniform velocity gradient of molecules approaching the wall. But in the present derivation the term is neglected, since the velocity gradient of molecules in transit between the wall and plane where they join the viscous phase is zero. However, since the definition of the viscous phase is different from that assumed by Kennard, the value of the slip coefficient derived is compared with previous values in § 4.2.

The slip term in equation (2) is incomplete since it does not vanish as the pressure approaches zero. But as only those molecules in the viscous phase can assume the slip velocity, the slip term must be multiplied by the fraction of molecules in the viscous phase compared with the total number of molecules. To find the fraction it is assumed that after a molecule has left a wall it joins the viscous phase when it collides with another molecule whose last collision was an intermolecular collision. The assumption may be justified by the fact that molecules possess viscous and slip velocities only if they have a mean velocity in the direction of flow and they only have such a velocity if their previous collisions were intermolecular collisions. For the assumption to be utilized, for all molecules leaving a wall, the average distance travelled by molecules when they move from one layer to another layer distance x away is first calculated using the equations due to Kennard (1938, pp. 60–4) and the average distance can be shown to be $2x$. Let us consider now a thin layer EF between two parallel plates of cross section ABCD as shown in figure 4. The number of molecules entering EF from above is $\frac{1}{2}n\bar{c}$ and similarly for molecules entering from below. But of the molecules entering EF from above and below, the number moving directly to EF from CD and AB without being involved in an intermolecular collision, by using the

result of the average distance travelled, is

$$\frac{1}{4} n\bar{c} \exp\left\{\frac{-2(h-x)}{\lambda}\right\} \quad \text{and} \quad \frac{1}{4} n\bar{c} \exp\left\{\frac{-2x}{\lambda}\right\}$$

respectively. Therefore the fraction of molecules in EF with their last collision having taken place at a wall is

$$\frac{1}{2} \left[\exp\left\{\frac{-2x}{\lambda}\right\} + \exp\left\{\frac{-2(h-x)}{\lambda}\right\} \right].$$

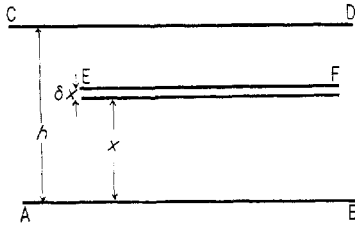


Figure 4. Channel cross section.

Now consider molecules leaving AB and moving towards CD. At a distance x from AB let the fraction of molecules still moving without having been in collision with another molecule, whose last collision was an intermolecular collision, be $f(x)$. But the fraction of the total molecules colliding in δx is $2\delta x/\lambda$; thus the fraction of molecules joining the viscous phase in δx is

$$f(x) - f(x + \delta x) = \frac{2\delta x f(x)}{\lambda} \left[1 - \frac{1}{2} \left\{ \exp\left\{\frac{-2x}{\lambda}\right\} + \exp\left\{\frac{-2(h-x)}{\lambda}\right\} \right\} \right].$$

By using Taylor's theorem, this becomes a differential equation which when solved becomes

$$\lg\{f(x)\} = \frac{1}{2} \exp\left\{\frac{-2(h-x)}{\lambda}\right\} - \frac{1}{2} \exp\left\{\frac{-2x}{\lambda}\right\} - \frac{2x}{\lambda} + C.$$

At the surface where x is zero, $f(x)$ must be unity; then

$$C = \frac{1}{2} - \frac{1}{2} \exp\left\{\frac{-2h}{\lambda}\right\}.$$

At any distance x from the wall the fraction of molecules joining the viscous phase in element δx is $-f'(x)\delta x$. Then the number of molecules in transit at any time between the element δx , where the molecules joined the viscous phase, and the wall, using both walls of the channel, is $-na(h-x)f'(x)\delta x$ where a is the wall surface area associated with δx . But the total number of molecules is nah ; thus the fraction of molecules in the viscous phase is

$$-\int_0^h \frac{h-x}{h} f'(x) dx \tag{3}$$

and the slip term becomes

$$f_s(\alpha) = -\int_0^h \frac{2(h-x)}{h} f'(x) dx. \tag{4}$$

The variation of $f_s(\alpha)$ with α , as calculated numerically by using a computer, is shown in figure 6.

4.2. The viscous term

As already explained, for low values of α all the molecules in a channel cannot support viscous flow; thus a correction is applied to the term $\alpha/6$ in equation (2). Scott and Dullien

(1962) and Fryer (1966) assumed that all the molecules, except those travelling on wall-to-wall collision paths, were in the viscous phase. A better definition of the viscous phase has been given in § 4.1 which will now be used to derive the viscous term. Further, since a molecule supports viscous velocities between the points where it joins and leaves the viscous phase and not between the channel walls, another correction is applied, this time to the width of the viscous phase.

Let us consider figure 4 and let molecules travelling from AB towards CD join the viscous phase at EF; thus these molecules are in the viscous phase between the plane EF and the wall CD. Therefore for these molecules the channel width as used in the viscous term should be reduced from h^3 to $(h-x)^3$. By using equation (3) the viscous term becomes

$$f_v(\alpha) = -\frac{\alpha}{6h^4} \int_0^h (h-x)^4 f'(x) dx. \quad (5)$$

This integral has been evaluated by numerical analysis and the variation of $f_v(\alpha)$ with α is shown in figure 6. The fractional difference between $f_v(\alpha)$ and $\alpha/6$ can be shown to become zero as α becomes large.

In § 4.1 the slip coefficient uncorrected for the fraction of molecules in the viscous phase was shown to be 2, which differs considerably from the value of $\frac{4}{3}$ used by Scott and Dullien (1962) and Fryer (1966), and from the value of unity derived by Kennard (1938). However, the value of the slip coefficient can be accurately compared with previous values only when the definition of the slip velocity and the reduced width of the viscous phase are taken into account. Let k be the slip coefficient as used by Knudsen (1909), Kennard (1938), Millikan (1923), Fryer (1966) and others; then the usual viscous flow with slip is given by

$$Q = \frac{-\beta b (\pi M)^{1/2}}{2 (2RT)} \left(\frac{h^2}{6\lambda} + hk \right) \frac{\delta\alpha}{\delta l}$$

when the fraction of molecules in the viscous phase has been neglected. But by using equations (4) and (5) and neglecting $f'(x)$, which represents the fraction of molecules in the viscous phase, the flow equation becomes

$$Q = \frac{-\beta b (\pi M)^{1/2}}{2 (2RT)} \left\{ \frac{(h-x)^4}{6\lambda h^2} + 2(h-x) \right\} \frac{\delta\alpha}{\delta l}$$

where x is defined as in § 4.1. For molecules leaving a wall it can be shown that if α is greater than 5 then, to within 0.5%, $x = \frac{4}{3}\lambda$, since under such conditions molecules on average join the viscous phase after leaving a wall with their second intermolecular collision. Thus it is shown that k is given by

$$k = \frac{\alpha}{6} \left(1 - \frac{4}{3\alpha} \right)^4 + 2 - \frac{8}{3\alpha} - \frac{\alpha}{6}$$

and is a function of α . The variation of k with α is shown in figure 5 and the values of k are in good agreement with most previous experimental results. Knudsen (1909) quoted a value of $k = 1.33$ for oxygen and carbon dioxide flowing through circular capillaries, but this is the value to be expected since α was between 2 and 8. Further, Millikan (1923) had determined the values of k from experiments on the deviation from Stokes's law for falling oil droplets, where α was of the order of 25, and except for Shellac surfaces the results are within the experimental errors to the value of 1.14 expected from figure 5. Because of the variation of k with α , the revised definition of the slip velocity, giving a slip coefficient of 2, will be used in the rest of the work, but it should be remembered that the variation in k with α is a direct result of using the revised definitions of the slip velocity and the viscous phase.

4.3. The diffusive term

During the discussion of the slip term it was assumed that the density gradient of molecules leaving a wall does not contribute to the flow. This is not strictly true since diffu-

sive flow occurs down the pressure gradient, which in the limit of α tending to zero becomes the pure molecular flow.

Equations representing diffusive flow through circular capillaries and between parallel plates have been derived by Hiby and Pahl (1952) by considering the effects on the flow of long molecular paths and anisotropic spatial distribution of collision partners. Unfortunately their equations are only valid for values of α less than 0.01, but the method which they used for circular capillaries has been extended by Roberts (1968) to cover all values of α for parallel-plate channels. A detailed derivation is long and thus will be published at a later date but the result is given in figure 6 and by the equation

$$Q_a = -\frac{bh\beta}{2} \left(\frac{\pi M}{2RT}\right)^{1/2} f_a(\alpha) \frac{\delta\alpha}{\delta l}$$

where

$$f_a(\alpha) = \frac{2}{\pi} \left\{ \frac{4}{3\alpha} - \frac{1}{2\alpha^2} + \frac{2F_1(\alpha)}{\alpha^2} + F_2(\alpha) \right\}.$$

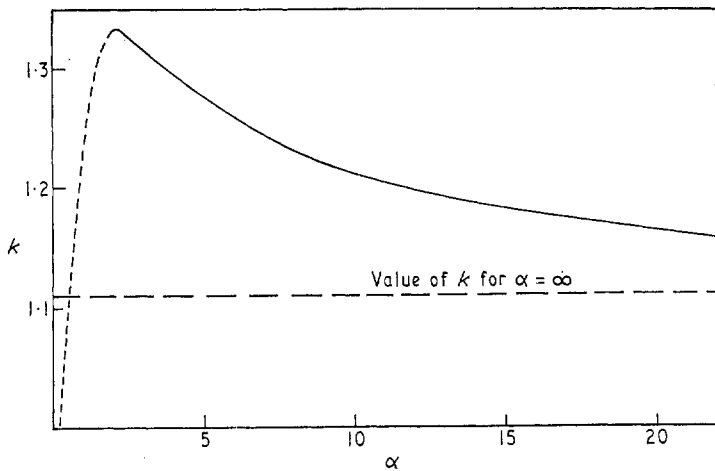


Figure 5. Variation of the slip coefficient k with α .

The first three terms are given in Hiby and Pahl's equation (3.8) where

$$F_1(\alpha) = \int_0^{\pi/2} \sin^3\beta \cos\beta \exp\left(\frac{-\alpha}{\cos\beta}\right) d\beta.$$

The $F_2(\alpha)$ term represents the additional flow due to anisotropic spatial scattering of collision partners and has been evaluated by Roberts (1968) by using a similar method to that used by Hiby and Pahl for circular capillaries and by solving a number of integrals with a computer. The term is a correction to the usual diffusive equation and is of the order of 20% of the other three terms in $f_a(\alpha)$. As in figure 6 $f_a(\alpha)$ tends to zero as α becomes large and tends to the value of $2(3 - 4 \lg\psi)/\pi$ given by Hiby and Pahl's equation (3.10) as α becomes small, where $\psi = 1.781$. This is not the same as Knudsen's equation but this is to be expected since von Smoluchowski (1910) has shown that Knudsen's equation is correct only for circular cross sections.

4.4. The flow equation

The flow equation at any cross section of the channel, where the mean pressure is much greater than the differential pressure, is given by the sum of the three flow terms; thus

$$Q = -\frac{bh\beta}{2} \left(\frac{\pi M}{2RT}\right)^{1/2} f(\alpha) \frac{\delta\alpha}{\delta l}. \tag{6}$$

The variations of $f(\alpha)$ and its components $f_v(\alpha)$, $f_s(\alpha)$ and $f_a(\alpha)$ with α are shown in figure 6, where a minimum in the specific flow is shown to be at $\alpha = 0.5$.

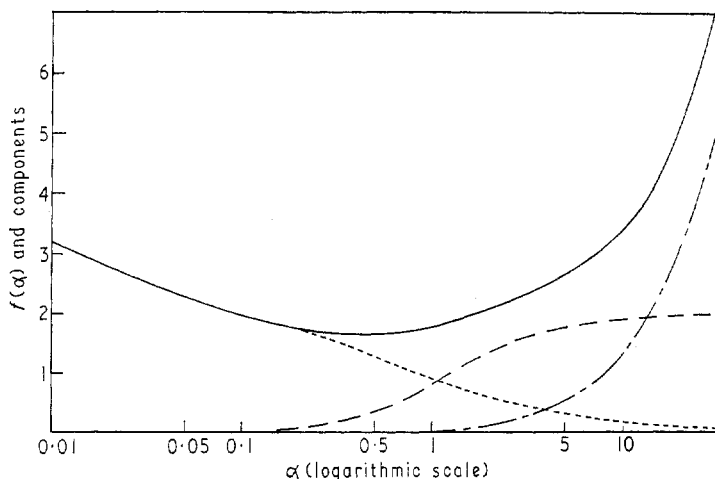


Figure 6. Variation of $f(\alpha)$ ———, $f_v(\alpha)$ — — — — —, $f_s(\alpha)$ - - - - and $f_a(\alpha)$ ····· with α .

But in the experiments carried out it was not possible to assume that the mean pressure was much greater than the differential pressure; thus equation (6) is integrated numerically by a computer between the limits of $\alpha = 0$ and $\alpha = \alpha_p$, since for all the experiments the pressure behind the channel could be assumed to be zero. The variations of $f(\alpha_p)$ and its components with α_p are shown in figure 7. The values of G for any gas are then given by

$$G = -h\beta \left(\frac{\pi}{2R} \right)^{1/2} f(\alpha_p) \quad (7)$$

and thus G is completely defined by h , β and α_p , or since $\alpha_p = h\Delta P/\beta$, G is defined by h , β and Δp .

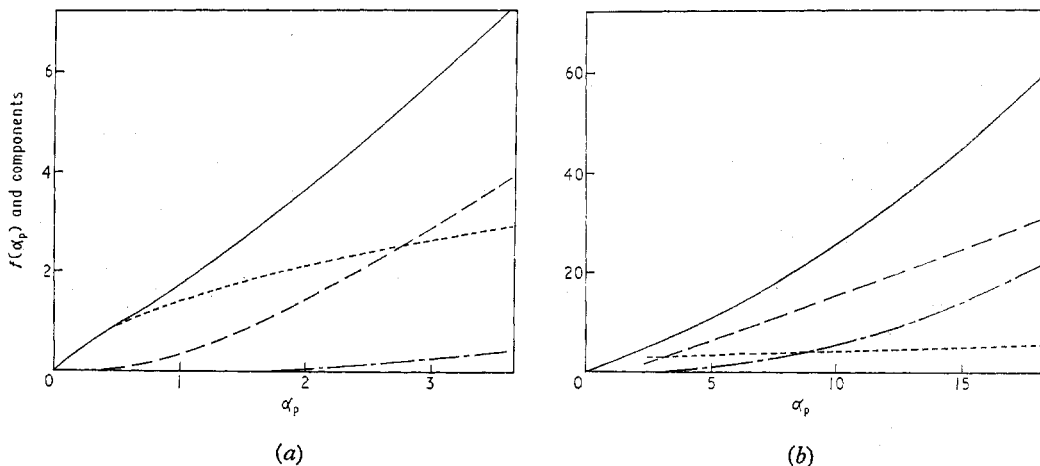


Figure 7. Variation of $f(\alpha_p)$ ———, $f_v(\alpha_p)$ — — — — —, $f_s(\alpha_p)$ - - - - and $f_a(\alpha_p)$ ····· with α_p .

5. Discussion

The experimental results for all the gases and vapours used agree with the theory to within the experimental error of 4% and are all represented by $f(\alpha_p)$ in figure 7. The parallel-plate channels used varied in width between $5.28 \mu\text{m}$ and $44.4 \mu\text{m}$ in the case of hydrogen and helium, and between $7.10 \mu\text{m}$ and $44.4 \mu\text{m}$ in the case of air, argon, carbon dioxide and water vapour. The differential pressure across the channel was in all cases between 5 torr and 15 torr; thus the range of α_p used was between 0.2 and 17.7. Table 1 shows the values of α_p for $\Delta P = 7.72$ torr for the greatest and the least channel widths used, together with the values of β for all the gases and vapours used, at a temperature of 20°C .

Table 1. Values of β and the range of α_p used at $\Delta P = 7.72$ torr

	Hydrogen	Helium	Air	Argon	Carbon dioxide	Water vapour
β (g s^{-2})	12.21	19.03	6.605	6.893	4.318	4.530
α_p (max)	3.3	2.4	6.9	6.6	10.6	10.1
α_p (min)	0.44	0.28	1.1	1.0	1.7	1.6

Owing to the agreement between the theory and the experimental results, equation (7) can be used to represent all the experimental results, thus showing that for the range of α_p used G is a function of h , β and α_p . But for a certain gas at a certain temperature β is a function of η , and α_p is a function of h , η and ΔP ; hence G is a function of h , η and ΔP only. Further, figure 7 is a universal curve whenever the pressure on the low-pressure side of the channel can be assumed to be zero.

One of the characteristics of past experiments on the flow of gases through artificial channels has been a minimum observed in the variation of the specific flow with pressure. The minimum was not observed directly in the experiments already described but the presence of a minimum is shown theoretically at $\alpha = 0.5$ (see figure 6) where the mean pressure in the channel is much greater than the differential pressure across the channel. However, for channels where the differential pressure is of the same order as the mean pressure the minimum becomes a point of inflection at $\alpha_p = 0.5$ which is shown in figure 7. The point of inflection is also shown from the experimental results given in figure 3(b) by the fact that the graph of values of G_p for helium, which are higher than those for carbon dioxide (and the other gases) for h less than $9 \mu\text{m}$, crosses the curve for carbon dioxide in the region of $h = 11 \mu\text{m}$. The crossing of the two curves arises theoretically from the fact that the point of inflection in the variation of G_p with h occurs at $h = 9.26 \mu\text{m}$ for helium and at $1.13 \mu\text{m}$ for carbon dioxide, owing to the different values of β , 19.03 and 4.03 respectively. Thus it is seen that at a certain channel cross section where $\alpha = 0.5$ a minimum in the specific flow occurs, but that this minimum is not observed experimentally when the differential pressure across the channel is greater than the mean pressure in the channel.

When the experimental results for water vapour were compared with those for the gases no detectable difference could be observed; thus any surface effects which might occur for water vapour did not contribute to the rates of flow measured. Although surface effects were not observed, it may still be possible to observe such effects for parallel-plate channels for smaller plate separation, but the apparatus used in the experiments described could not be extended to measure such low rates of flow.

Further work is now being planned on the flow of gases and vapours through finer channels of the order of $0.5 \mu\text{m}$ and at small differential pressures.

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